The Stock Market

Rough Draft Due Wednesday, April 22

For each of the milestones, please neatly type up your responses. Using LaTeX or Overleaf is a plus, but if you can get to it look nice in Word or OpenOffice that is also fine. Your group is only responsible for one write-up at each milestone. You will eventually compile these into a single, cohesive write-up with Introduction, Model, Results, Conclusion, and References sections.

Introduction

The stock market is notoriously hard to predict (or else we'd all be rich!). We can attempt to model it using a **stochastic** model, which has randomness built in. Unlike the **deterministic** models we've studied thus far, which will give the same results for the same parameters every time you run them, stochastic models will give a different, random result each time. We can still model and predict things like their average behavior and variance however.

Start with the simplest possible model of how a stock's price changes day to day. Consider X_n to be the closing price of a stock at day n. X_n is a **random variable**, which we often denote with a capital letter. This means we cannot determine its value ahead of time, but we can describe its distribution. In this project we will consider several distributions and try and match the best one to data.

Model

Consider X_{n+1} to be a random variable that depends on X_n. The simplest place to start is to consider that X_n goes up or down by a fixed amount a each day. We can describe the conditional distribution of X_{n+1} given X_n as

$$p_{X_{n+1}|X_n}(x_{n+1}|x_n) = P\left(X_{n+1} = x_{n+1}|X_n = x_n\right)$$

where the right-hand side represents the conditional probability that $X_{n+1} = x_{n+1}$, given $X_n = x_n$. Note that we use capital letters for random variables and lower-case letters for their realizations. For example, if Y represents the number on a six-sided die roll, then we can state $P(Y = y) = \frac{1}{6},\ y = 1, 2, \ldots, 6$.

Milestone 1

Due 2/10:

1. According to the above, if there is a probability p that the price goes up by a each day, argue that

$$p_{X_{n+1}|X_n}(x_{n+1}|x_n) = \begin{cases} p, & x_{n+1} = x_n + a \\ 1-p, & x_{n+1} = x_n - a \\ 0, & \text{otherwise} \end{cases}$$

2. Construct a difference equation model using a random variable A for the change in price between day n and $n+1$. Determine the probability mass function of A, $p_A(x) = P\left(A = x\right)$

3. What are the variables and parameters of this model?

4. Simulate the system in Excel or Octave using random numbers.

For Rough Draft

Due 4/22:

1. Try simulating in Octave by starting with the following: `t = 0:100; x0 = 1;` and a for loop, e.g., `for i = 1:99; x(i+1) = x(i) + rand; end;`. This will be a very basic starting place and you can then look at different distributions.

2. Incorporate p in your simulation by using `if/then` statements like `if rand < p; x(i+1) = x(i) + a; elseif rand >= p; x(i+1) = x(i)-a; endif`

3. Try running several times to see the variance in results

4. Experiment with different distributions for your random increase/decrease, such as `normrnd`, `unifrnd`, `exprnd`, etc.

5. Try to see which models best fit data